Discussion

In free vibration, a mechanical system is excited by an initial impulse, then allowed to vibrate freely without any further force interaction. A system in free vibration oscillates with its natural frequency and gradually settle down to zero due to damping effects. In forced vibration, an external force is supplied to the system. In an undamped vibration no energy is lost or dissipated during oscillation, however in a damped vibration some energy is dissipated as heat or sound.

Free vibration response

In the free vibration, no external force is applied to the mass.

Equilibrium position of the mass is: $X = A\, sin\sqrt{\frac{k}{m}}\, t + B\, cos\sqrt{\frac{k}{m}}\, t$

Natural frequency is the frequency at which a system oscillates, in the absence of any driving or damping force. However, forced frequency is the frequency of a system vibrating with applied force. The case at which forced frequency is equal to the natural frequency is called resonance.

Where $\sqrt{\frac{k}{m}}$ is the angular natural frequency: $\omega_n = \sqrt{\frac{k}{m}}$

The *period* of the oscillation is the time taken for one complete cycle: $\tau = \frac{1}{f} = \frac{2\pi}{\omega_n}$

The frequency of vibration: $f = \frac{1}{\tau} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

The logarithmic decrement (δ) is the natural log of the ratio of the amplitudes of any two successive peaks : $\delta = \ln(\frac{x_i}{x_{i+1}})$

Where x_i and x_{i+1} are any two successive amplitudes.

Logarithmic decrement for harmonic motion can be obtained from displacement, velocity or acceleration measurements, as the ratio between cycles remains the same.

The damping ratio (ζ) is an expression of the level of damping in a system relative to the critical damping, it describes how rapidly oscillations in a system decay after a disturbance. It can be calculated dividing the actual damping by the critical damping.

$$\zeta = \frac{c}{c_c}$$

when $\zeta<0.05$, the damping ratio and logarithmic decrement are related in the following equation:

$$\zeta = \frac{\delta}{2\pi}$$

The natural frequency of the *undamped* system : $\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}$

$$\omega_d = \frac{2\pi}{\tau_d}$$

Damped natural frequency is related to the undamped natural frequency by the following formula:

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

The damped natural frequency is less than the undamped natural frequency, and for most cases the damping ratio is relatively small and is neglected.

the undamped natural frequency: $\frac{c_c}{2m} = \sqrt{\frac{k}{m}} = \omega_n$

$$c_c = 2m\omega_n$$

The actual damping in a system in term of damping ratio ζ

Forced vibration resulting from an eccentric mass

When a viscous damper is added to the beam, external force is applied. And energy is dissipated by viscous damping (due to the fluid's resistance).

The vertical force produced on mass by the rotation of mass at any instant during the vibration: $F = MR\omega^2\cos(\omega t)$

The amplitude of the steady state response : $X = \frac{MR\omega^2}{K-m\omega^2+j\omega c}$

$$X = (\frac{MR}{m})\frac{r^2}{1-r^2+2jr\zeta}\ ,\ \ r = \frac{\omega}{\omega_n}$$

Damping affects the response near the natural frequency ,the response is max at resonance when: $\omega_r = \frac{\omega_n}{\sqrt{1-2\zeta^2}}$

ω_r is resonance frequency.